CMCDS Data Exploration and Trading Strategies

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This Draft: December 13, 2007
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Abstract

This paper explores trading strategies to identify possible imbalances that may have been existed in the credit markets, during the period 2001–2006, when pairing CDS and CMCDS on the same name. To this end, a large database of single-name CDS premia is used to produce the corresponding CMCDS prices, derived by implementing common market models. It appears that, in general, it would have been more profitable to sell CDS and to buy CMCDS, since at least 85% of the names analysed had a negative cumulative net trading profit/loss over the 5 years period considered.

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Credit derivatives are derivative securities whose payoff is conditioned on the occurrence of a credit event. Credit events typically include bankruptcy, failure to pay, default or restructuring. In practice credit derivatives widely used include total return swaps, spread options, and credit default swaps[1], but new product emerge every year. Credit default swap (CDS) established itself as one of the most liquid and important instrument, accounting for around a third of the credit market as at the first quarter of 2006 (BBA(2006)). They provide insurance against the risk of a default by a particular company. The purpose credit default swaps is to allow credit risks to be traded and managed in much the same way as market risks. The premium (called the CDS spread) in a CDS spread contract is determined by matching the discounted cash flows of a fixed leg paid by the protection buyer and a loss leg which corresponds to the net payment made by the protection seller to the protection buyer in case of default. As discussed in more detail in Duffie (1999) and Hull and White (2000), the credit default swap spread should be very close to the credit spread of a par yield bond issued by the reference entity over the par yield risk-free rate. This can be shown using a no arbitrage argument. Buying a par yield bond and a CDS on the reference entity an investor eliminates almost all the credit risk associated with default on the bond. This means that, denoting with y the yield on a T-year par yield bond issued by a reference entity, with r the yield on a T-year par yield riskless bond, and with S the T-year CDS spread (i.e. S is the periodical premium paid by the the protection buyer), the relationship $S = y - r$ should hold. In fact, if S is less than $y - r$, buying a corporate bond and the credit default swap and short selling a riskless bond will result in an arbitrage. If S is greater than $y - r$, then an arbitrageur will find it profitable to short a corporate bond, sell the credit default swap, and buy a riskless bond[2].
The validity of the theoretical equivalence of CDS spreads and credit bond yield spreads is tested in Blanco et al. (2005). Using a dataset of 33 U.S. and European investment-grade firms it is found that this parity relation holds on average over time for most companies, suggesting that the bond and CDS markets may price credit risk equally. Deviation from parity are found only for three European firms, for which the authors find that CDS prices are substantially higher than credit spreads for long periods of time. These cases are attributed to a combination of both imperfections in the contract specification of CDSs and measurement errors in computing the credit spread. For all the other companies they find only short-lived deviations from parity in the sample. This is because CDS prices lead credit spreads in the price discovery process, meaning that the CDS market leads the bond market in determining the price of credit risk. The relationship between credit default swaps and corporate spreads is investigated also in Longstaff et al. (2005). After developing closed-form expressions for corporate bond prices and credit default swap premium within the familiar Duffie and Singleton (1997, 1999) framework, they use the information in credit default swaps to obtain direct measures of the size of the default and nondefault components in corporate spreads. In other words they investigate what proportion of corporate yield spreads is directly attributable to default risk and how much of the spread depend on other factors such as liquidity and taxes. To answer these questions they use the information in credit default swap premia to provide direct measures of the size of the default and nondefault components in corporate yield spreads. Using CDS premia for 5-year contracts and the corresponding corporate bond prices for 68 firms traded during the period March 2001–October 2002, they find that the default component represents the majority of corporate spreads, accounting for more than 50% of the total corporate spread, even for the highest-rated

[1] For an overview of credit derivatives see Tavakoli (1998); Schönbucher (2003)

[2] Some assumptions and approximations are to be made in this arbitrage argument (see Hull et al., 2004).

investment-grade firms. Also the nondefault component is found to have a significant impact on corporate spreads. In particular, the nondefault component is time varying and mean reverts rapidly and can be explained by measures of bond-specific illiquidity such as the bidask spread and the outstanding principal amount. Finally, taxes don't seem to play an important role in explaining the nondefault component. Norden and Weber (2004) apply traditional event study methodology is applied to examine whether and to what extent stock and CDS markets responded to rating announcements during the years 2000-2002. Rating announcement events are collected from the three major rating agencies (Standard & Poor's, Moody's and Fitch) to find that CDS markets anticipate rating downgrades and that anticipation starts approximately 60-90 days before the announcement day. This result is consistent with Hull et al. (2004), in which credit default swap changes conditional on a ratings announcement are examined. Reviews for downgrade are found to contain significant information. However this is not the case when downgrades and negative outlooks are considered. The main conclusion is that significantly positive CDS spread changes happen before negative rating events, but positive rating events are much less significant. The latter conclusion, however, even though consistent with the results of studies on the relationship between rating events and bond yields, may be influenced by the limited number of positive rating events in the sample studied. An investigation of the US corporate credit default swap market is provided in Schneider et al. (2007). The paper, in order to explain both the cross-section and the time series of CDS premia and to accomodate simultaneous as well as individual jumps in the risk-free and credit-risky state variables, proposes a three-factor observable jump-diffusion model for the riskless short rate and a two-factor jump-diffusion model for the default intensity of an obligor. The stochastic long-run mean of the intensity, as well as the default intensity itself, are found to be extremely persistent. The number of estimated jump events in the credit-risky components per year ranges between two and fifteen. Furthermore, the number of jumps in credit spreads is bigger for lower rated obligors. The number of jumps in the risk-free term structure is estimated in approximately four per year. As far as simultaneous jumps of the risk-free and the credit-risky components are concerned, their number is found to be approximately 0.3 per year. Pan and Singleton (2008) use a full term structure of sovereign CDS spreads to derive the market-implied default intensity and also the implicit loss rate. They argue that a lognormal process for the default intensity (as opposed to a square-root process used in Longstaff et al., 2005) is capable of capturing most of the variation in the term structure of spreads. The data used in the paper involves three countries (Mexico, Turkey, and Korea) with different credit ratings and covers the period March 2001–August 2006. Using MLE, the risk-neutral default intensity are found to be highly persistent and significant differences are found between the parameters in the process for the default intensity under the risk-neutral and the historical measures. This implies substantial market risk premia due to unpredictable changes in the default intensity. Furthermore, it is found for the 5-year maturity that these risk premia can be explained by measures of global risk, financial market volatility and macroeconomic policy, such as the CBOE VIX volatility index, the spread between the US Industrial 10-year BBB Yield and the 6-month Treasury bill yield and the own-country implied currency option volatility. Berndt et al. (2005) assume a lognormal model for the default intensity using a dataset of corporate CDS spreads from three sectors (broadcasting and entertainment, healthcare, and oil and gas) for the period 2000–2004. The main conclusions of the paper are that 5-year Moody's KMV EDFs explain over 74% of the variation in 5-year CDS rates across issuers and time, and that risk premia changed dramatically over time, from peaks in the third quarter of

2002, to a significant decrease at the end of 2003. In order to derive the credit risk premia, the vector of parameters governing the evolution of the default intensity process under the actual probability measure is estimated from the EDF data, whereas the vector of parameter governing the risk-neutral intensity process is estimated from 1-year and 5-year CDS rates and from 1-year EDFs.

By analogy with an interest rate swap and the constant maturity swap (CMS) contract, another traded credit derivative is the constant maturity credit default swap (CMCDS). By entering such a contract the buyer pays a premium (spread) in exchange of protection, exactly like in CDS contract. The essential difference between a CMCDS and CDS arises in the payment leg: while in a CDS the spread is fixed, in a CMCDS the spread is floating and calculated according to an indexing mechanism. In particular the spread is set equal to the prevailing reference CDS spread at each reset date times a factor known as the participation rate (PR). As a consequence, in a CMCDS contract the loss leg is paired with a floating leg, where spread payments are indexed against a reference constant maturity CDS spread at each reset date. Floating cash flows are linked to a constant-maturity term that goes under the name of *constant maturity tenor*. The reference constant maturity CDS spread does not have to have the same nominal maturity as the maturity of the contract itself. Hence one could trade a 5-year CMCDS referenced by the 3-year or 7-year CDS spread.

The main attraction of CMCDS is that, similar to constant maturity swaps (CMS) in fixed income market, they allow investors to take curve views and, when they are combined with a CDS position, they give the investor the capability to express views on credit spreads with no default risk exposure. As a matter of fact, a short CMCDS long CDS position allows investors to isolate spread risk (i.e. the risk of changes in the premium not related to an actual credit event) and to hedge default risk. Also, CMCDS are useful for protection sellers to hedge against spread widening risk. Recall that in a CDS contract the protection seller is bound to receive a constant premium until maturity or until the occurrence of a credit event. However, if before maturity CDS spreads increase, implying that protection has become more expensive, he or she still receives the spread agreed upon at the beginning of the contract, even though the market conditions are changed. If the protection seller is worried by such a possibility, a reasonable solution could be a short position in a CMCDS contract. The investor could also benefit from the fact that the CMCDS contract has a lower mark to market than a similar CDS contract when spreads widen.

The scope of this study is to identify possible imbalances that may exist in the credit markets when pairing CDS and CMCDS on the same name. The general idea is to form a swap trading strategy whereby a fixed premium payment is netted against a floating one, both representing protection premia against default. This strategy has the advantage that default risk is eliminated and only counterparty risk is taken. A large database of single-name CDS premia has been used to produce the corresponding CMCDS prices using common market models. In doing this we use the full term structure of CDS spreads to infer default information. This work differs from Pan and Singleton (2008) in that we include in our analysis corporate CDS. The CDS market is thus analysed in its completeness and the existence of arbitrage strategies based on the swap CDS-CMCDS is investigated. With this in mind, single-name CDS spread data is used to replicate as much as possible the *would have been* CMCDS spreads. Next, the possible paired trading strategy of going long a CDS and short a CMCDS is constructed and the corresponding profit-loss profile across the database is analysed. The applied literature on CDS so far has focused either on issues like the validity of the theoretical equivalence of

CDS prices and credit bond spreads or the determinants of credit default swap changes. In this paper, instead, using a dataset of CDS spreads that is large both in terms of the cross section of obligors included and in terms of the period covered, we try to identify, by the means of a statistical arbitrage analysis, trading strategies which employ CDS and CMCDS.

In order to construct the trading strategy mentioned above, of course the CMCDS prices implied in the CDS quotes must be produced. This requires, first of all, that we bootstrap the survival probabilities from the observed CDS quotes. To this end both nonparametric (piecewise constant hazard rates) and parametric (Nelson-Siegel interpolation and an OU process for the hazard rates) methods commonly used in practice have been implemented. To derive the survival probability from the CDS quotes, also a for pricing CDS contracts is required. As far as the methodology for the pricing of CDS and CMCDS contracts, we employ is the one used by many banks, precisely because our aim is to compute the profits (or losses) a real bank could have made by implementing the CDS-CMCDS swap during the period considered.

The remainder of this paper is thus organized as follows. The next section describes the pricing of CDS and CMCDS. For the latter, the focus is on the convexity adjustment. The next two subsections present different methodologies to back out the hazard rates and hence the survival probabilities from CDS spreads, the details of how the pricing of CMCDS is done practically and how the discount factors are calculated. A recap of the algorithm to be applied each day and for each obligor when the trading strategies are implemented, is presented in section II along with some numerical examples regarding the survival probability bootstrapping. The results of the statistical arbitrage analysis are reported in section III. The final section concludes.

I Market Models for CDS and CMCDS Pricing

A Pricing Credit Default Swaps

In this section we sketch the framework for the pricing of CDS, since this also contributes to the pricing of CMCDS.

The expanding literature on the pricing of credit default swaps includes among many others Ben Ameur et al. (2006); Bielecki et al. (2005); Chu and Kwok (2003); O'Kane and Turnbull (2003). For the valuation of a CDS contract when the payoff is contingent on default by a single reference entity the methodology of Hull and White (2000) is very popular.

Consider a CDS contract with periodic premium S to be paid at times $s_1 < s_2 < \ldots < s_N = T$ or until default, in exchange for a single protection payment to be made at the default time τ, provided that $\tau \in (s_0, s_N]$. Denoting by R the recovery rate and assuming that the loss given default equals $1 - R$, the CDS discounted value at time t for the protection seller is[3]

$$\Pi(t) = DF(t,\tau)(\tau - s_{\beta(\tau)-1}) \times S \times \mathbb{1}_{\{s_0<\tau<s_N\}}$$
$$+ \sum_{i=1}^{N} \Delta(s_i, s_{i-1}) \times DF(t, s_i) \times S \times \mathbb{1}_{\{\tau \geq s_i\}} - \mathbb{1}_{\{s_0<\tau\leq s_N\}} DF(t,\tau)(1-R)$$

where $s_{\beta(\tau)}$ is the first date among the s_i's that follows τ, $DF(t_a, t_b)$ is the stochastic discount factor at time t_a for maturity t_b and $\Delta(s_i, s_{i-1})$ is the year fraction between s_{i-1} and s_i. Given

[3] $\mathbb{1}_{\{.\}}$ denotes the indicator function.

the filtration $\mathcal{G}_t$ which represents the information flow of interest rates, default intensities and includes default monitoring, the price of the contract is then computed according to risk-neutral valuation, i.e. $\text{CDS}_{0,N}(t,S,R) = \mathbb{E}(\Pi(t)|\mathcal{G}_t)$. The periodic premium for the protection buyer entering such a contract at time 0 is defined to be the S that makes the CDS value equal to zero at time 0: $\text{CDS}_{0,N}(0,S,R) = 0$. In the remainder of the paper the premium to be paid when entering such a contract will be denoted by $S(0,T)$. Let θ_t be the risk neutral default probability density at time t, so that the probability of default in $[0,T]$ is $\int_0^T \theta_t \mathrm{d}t$. Clearly, the probability that no credit event occurs up to time t is $\pi_t = 1 - \int_0^t \theta_u \mathrm{d}u$. Consequently the periodic premium to be paid by the buyer of the CDS when the risk-free rate is constant and equal to r is

$$S(0,T) = \frac{(1-R)\int_0^T \mathrm{e}^{-ru}\theta_u \mathrm{d}u}{\sum_{i=1}^N \Delta(s_i, s_{i-1})\pi_{s_i}\mathrm{e}^{-rs_i} + \int_0^T a_u \mathrm{e}^{-ru}\theta_u \mathrm{d}u} \tag{1}$$

where a_u is the accrual payment at time u. Each term appearing in the summation at the denominator is the discounted present value of the expected payments made at time s_i, provided the reference entity survives until s_i. The second term represents the present value of the accrual payments. The numerator is the expected present value under the risk-neutral measure of the payoff received by the protection buyer.

A more formal framework for valuation of single-name credit derivatives (including credit default swaps and swaptions) is given in Jamshidian (2004) in which the general subfiltration approach of Jeanblanc and Rutkowski (2000) to modelling default risk, which includes the Cox-process setting of Lando (1998), is integrated with a numéraire invariant approach.

The model presented so far is quite elegant but in order to be applied one has to approximate somehow the integrals in (1). In this paper we follow what many banks do in practice to price CDS. The widely employed discretized valuation formula can be found in standard textbooks like Lando (2004, chap. 8) or Schönbucher (2003, chap. 3), Arvanitis and Gregory (2001) and Cherubini et al. (2004). In the contract described early in this section, the loss leg is

$$(1-R)\int_0^T DF(t)SP(t)\lambda(t)\mathrm{d}t$$

where R is the recovery rate, λ is the default intensity, $DF(t)$ denotes the discount factor for time t as seen at time 0 and $SP(t)$ denotes the survival probability for time t. The discount curve $\{DF(t)\}_{t\geq 0}$ can be easily derived from Libor-Swap rates, whereas the sequence of survival probabilities $\{SP(s_i)\}_{i=0,1,\ldots}$ can be bootstrapped from the term structure of CDS spreads (see subsection C).

The integral that appears in the formula for the loss leg can be approximated by

$$(1-R)\sum_j DF(u_j)[SP(u_{j-1}) - SP(u_j)]$$

where $\{u_1, u_2, \ldots\}$ is a partition (fine enough) of the interval $(0,T)$. Typically using a monthly grid results in a good approximation.

The payment leg is given by

$$S(0,T)\left(\sum_{i=1}^N \Delta(s_i, s_{i-1})DF(s_i)SP(s_i) + \sum_{i=1}^N \int_{s_{i-1}}^{s_i} \Delta(s, s_{i-1})DF(s)SP(s)\lambda(s)\mathrm{d}s\right)$$

where the second term is the accrual payment between s_{i-1} and s_i and Δ denotes the time accrual exposure with an Actual/360 convention. If we assume that the default will arrive on average in the middle of the interval, the second term can be approximated with

$$S(0,T)\sum_{i=1}^{N}\Delta(s_i,s_{i-1})DF(s_i)\frac{1}{2}[SP(s_{i-1})-SP(s_i)].$$

Thus the fixed payment leg can be calculated as

$$S(0,T)\sum_{i=1}^{N}\Delta(s_i,s_{i-1})DF(s_i)\frac{1}{2}[SP(s_{i-1})+SP(s_i)]$$

The credit default swap expressed in basis point at time 0 is easily calculated by equating loss and payment legs:

$$S(0,T)=\frac{(1-R)\sum_j DF(u_j)[SP(u_{j-1})-SP(u_j)]}{\sum_{i=1}^{N}\Delta(s_i,s_{i-1})DF(s_i)\frac{1}{2}[SP(s_{i-1})+SP(s_i)]}.$$

B Pricing Constant Maturity Credit Default Swaps

In this section we discuss how to price CMCDS in the framework presented in the last part of subsection A. Once again we present only the formula to be used in our computations. A more formal pricing framework, however, can be found in Brigo (2005), in which an approximated no-arbitrage market valuation formula for CMCDS is derived. The formula for CMCDS derived in Brigo (2005) is the analogous of the formula for constant maturity swaps in the default free swap market under the Libor market model. Closed-form solutions for Constant Maturity Credit Default Swaps, as well as Credit Default Swaps and Credit Default Swaptions, are derived also in Krekel and Wenzel (2006), where a Libor market model with default risk is used. Further details on CMCDS pricing can be found in Rajan et al. (2007) and in Brigo and Mercurio (2006).

The participation rate (PR) is the driver of the CMCDS premium and its value is strictly related to the slope of the CDS curve but not to the level. A participation rate not exceeding 100%, reflects the fact that the CDS curve is upward sloping. On the other hand the participation rate can be bigger than 100%, indicating a downward slope for the term structure of CDS spreads.

To derive the participation rate, we simply exploit the fact that since the loss leg from a CMCDS is identical to the loss leg from a CDS on the same obligor and same maturity, the fixed payment legs must be identical too. Hence, when the reference CDS has maturity m,

$$\begin{aligned}\mathrm{PR}\sum_{i=1}^{N}\mathbb{E}_0[S(s_{i-1},s_{i-1}+m)]\Delta(s_i,s_{i-1})DF(s_i)\frac{1}{2}[SP(s_{i-1})+SP(s_i)]\\=S(0,T)\sum_{i=1}^{N}\Delta(s_i,s_{i-1})DF(s_i)\frac{1}{2}[SP(s_{i-1})+SP(s_i)].\end{aligned}$$

Therefore, denoting by $d(s_i)=\Delta(s_i,s_{i-1})DF(s_i)$ we find

$$\mathrm{PR}=\frac{S(0,T)\sum_{i=1}^{N}d(s_i)[SP(s_{i-1})+SP(s_i)]}{\sum_{i=1}^{N}\mathbb{E}_0[S(s_{i-1},s_{i-1}+m)]d(s_i)[SP(s_{i-1})+SP(s_i)]}. \tag{2}$$

The major issue related to the previous formula is the evaluation of the expected value of future spreads in the denominator. It is clear that, when spreads evolve in a completely deterministic setting, future realised spreads will be completely determined from today's spread curve and thus the expected value equals the corresponding forward spread. However for high volatility names or long maturities a convexity adjustment is required.

B.1 The Forward CDS Spread

A long position in a forward default swap gives a credit protection that is active for a period of time in the future at a premium agreed upon today, but paid only during the active period of the contract.

The price for a forward contract for default protection during the time period $(t, t+m)$ is derived in Berd (2003):

$$FS(t, t+m) = \frac{S(0, t+m) - \delta(t, t+m)S(0, t)}{1 - \delta(t, t+m)} \tag{3}$$

where

$$\delta(t, t+m) \equiv \frac{\text{RiskyPV01}(0, t)}{\text{RiskyPV01}(0, t+m)}$$

and r_t and λ_t are the risk-free rate and the hazard rate respectively.

As mentioned before, the usual discrepancy between the realised future rate and the current forward rate is attributed to a convexity effect. This adjustment, important mainly for long maturity contracts, is discussed in the next subsection.

B.2 The Convexity Adjustment

The convexity adjustment is a common issue in mathematical finance especially in interest rate derivatives pricing (see Pelsser, 2003; Benhamou, 2000, 2002; Henrard, 2005a,b). Also in CMCDS pricing the convexity adjustment could play an important role, especially for long maturities and when the volatility is large. It is relatively easy to include a convexity adjustment when the default intensity, as commonly assumed, is described by a Ornstein-Uhlenbeck (OU) process:

$$\mathrm{d}\lambda_t = (k - \alpha\lambda_t)\mathrm{d}t + \sigma \mathrm{d}B_t. \tag{4}$$

Under (4) an approximated formula for the expected value of the future spread is derived in Calamaro and Nassar (2004):

$$\mathbb{E}_0[S(s_i, s_i + m)] \approx FS(s_i, s_i + m) + \frac{1}{2}\sigma^2 C_i[FS(s_i, s_i + m) - S(0, m)] \tag{5}$$

with

$$C_i = \frac{1 - \mathrm{e}^{-\alpha s_i}}{k\alpha}.$$

Using the above results we can write (2) as

$$\text{PR} = \frac{S(0, T)}{\overline{FS}(0, T) + \frac{\sigma^2}{2}\frac{C(0,T)}{D(0,T)}} \tag{6}$$

where

$$D(0,T) = \sum_{i=1}^{n} d(s_i)\frac{1}{2}[SP(s_{i-1}) + SP(s_i)]$$

$$C(0,T) = \sum_{i=1}^{n} d(s_i)\frac{1}{2}[SP(s_{i-1}) + SP(s_i)]C_i[FS(s_{i-1}, s_{i-1}+m) - S(0,m)]$$

and $\overline{FS}(0,T)$ is a weighted average of the forward CDS spreads over the reset dates:

$$\overline{FS}(0,T) = \frac{\sum_{i=1}^{n} d(s_i)[SP(s_{i-1}) + SP(s_i)]FS(s_{i-1}, s_{i-1}+m)}{\sum_{i=1}^{n} d(s_i)[SP(s_{i-1}) + SP(s_i)]}.$$

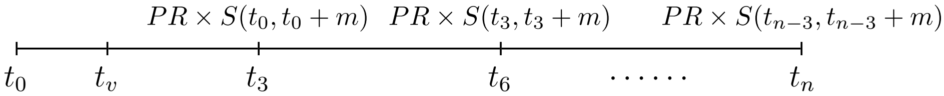
Equation (6) suggests an approximated formula for the participation rate of a CMCDS with maturity T and with constant maturity tenor m:

$$\text{PR} = \frac{S(0,T)}{\frac{1}{n}\sum_{i=1}^{n} FS(s_{i-1}, s_{i-1}+m)} \tag{7}$$

C Bootstrapping Survival Probabilities

Suppose we have a Libor discount curve $\{DF(t)\}_{t\geq 0}$ and the sequence of survival probabilities $\{SP(t_i)\}_{i=0,1,\ldots}$. The pricing time grid is described by n monthly periods given by $t_0 < t_1 < t_2 < \ldots < t_n = T$, with $t_i = i/12$ for all $i \in \{1, 2, \ldots, n\}$ and T denoting the maturity of the CMCDS contract. The contract is traded initially at time $t_v \in [t_0, t_3)$. The schedule of fixed payments is quarterly as this is the dominating market standard. The number of quarters fitting into the pricing time grid until maturity T is equal to $k = \left[\frac{n}{3}\right]$. It is evident that $k = \frac{n}{3}$ only if $t_v = t_0 \equiv 0$. The first premium is paid at time t_{n-3k+3} (which coincides with t_3 when n is a multiple of 3). A cash flow diagram is reported in Figure 1.

Figure 1: Cash flow diagram for constant maturity credit default swaps



Hence the participation rate when $t_v \equiv 0$ and the reference CDS has maturity m months is

$$\text{PR} = \frac{S(0,T)\sum_{j=0}^{k-1}\Delta(t_{n-3j}, t_{n-3(j+1)})DF(t_{n-3j})[SP(t_{n-3(j+1)}) + SP(t_{n-3j})]}{\sum_{j=0}^{k-1}\mathbb{E}_0[S(t_{n-3(j+1)}, t_{n-3(j+1)} + m)]\Delta(t_{n-3j}, t_{n-3(j+1)})DF(t_{n-3j})[SP(t_{n-3(j+1)}) + SP(t_{n-3j})]}. \tag{8}$$

where the expectation depends on the forward CDS (3).

Some care is needed in calculating the first coupon. There are some differences caused by whether the current settling day is within a month of the day of the first calendar coupon day or not. Again, t_v is the current valuation day and t_0 is the calendar coupon payment day prior to t_v.

If $t_3 - t_v > 1mth$ then[4] the first coupon is equal to $S(0,T) \times \Delta(t_3, t_v)$ and it is going to be paid at t_3 so it is going to be discounted by $\frac{1}{2}DF(t_3)[1 + SP(t_3)]$.

If $t_3 - t_v \leq 1mth$ then the first payment is delayed until the next coupon date and is coupled with a full coupon for the period. Thus the first coupon is equal to $S(0,T) \times [1 + \Delta(t_3, t_v)]$ and it is going to be paid at t_6 so it is going to be discounted by $\frac{1}{2}DF(t_6)[SP(t_3) + SP(t_6)]$.

Once survival probability curve is available, in order for this procedure to be implemented, a discount curve and the RiskyPV01 are needed as inputs. In particular, the latter calculation is done taking into account a pro-rata payment that arises from the fact that $n \geq 3k = 3\left[\frac{n}{3}\right]$. The RiskyPV01 is thus given by

$$\begin{aligned} \text{RiskyPV01}(0, t_n) = \frac{1bp}{2}\Big\{ & \Delta_0 \times DF(t_1) \times [1 + SP(t_1)] \\ & + \sum_{j=0}^{k-1} \Delta(t_{n-3j}, t_{n-3(j+1)}) DF(t_{n-3j})[SP(t_{n-3(j+1)}) + SP(t_{n-3j})] \Big\} \end{aligned} \tag{9}$$

where

$$\Delta_0 = \left[\frac{t_1 - t_v}{t_1 - t_0}\right]_{\text{ACT/360}}$$

and $[j/k]_{\text{ACT/360}}$ is the date fraction j/k converted into an ACT/360 convention.

As far as the discount factors are concerned, the standard choice for the risk-free rate is the Libor rates up to one year and swap rates after one year up to 10 years. When computing the discount factors it is important to recall that Libor rates over one week are spot rates whereas swap rates are par rates so that the calculation of discount factors from swap rates has to be done via bootstrapping. The main problem in calculating the discount factors is the calculation at intermediary points between the tenors of Libor or swap rates. Suppose we know the value of the risk-free discount factors for $0 = \mathcal{T}_0 < \mathcal{T}_1 < \ldots \mathcal{T}_r$ A very popular way to obtain the intermediary discount factors is to use log-linear interpolation. The discount factor for $t \in [\mathcal{T}_j, \mathcal{T}_{j+1}]$, $DF(t)$ will be given by

$$\log\left(DF(t)\right) = \frac{\mathcal{T}_{j+1} - t}{\mathcal{T}_{j+1} - \mathcal{T}_j} \log\left(DF(\mathcal{T}_j)\right) + \frac{t - \mathcal{T}_j}{\mathcal{T}_{j+1} - \mathcal{T}_j} \log\left(DF(\mathcal{T}_{j+1})\right).$$

The most delicate issue related to CMCDS pricing is the derivation of the survival probabilities. In this section we present a number of techniques, both parametric and nonparametric, commonly used in practise to infer survival probabilities from CDS market quotes. The three methods we focus are described in textbooks like (Brigo and Mercurio, 2006) or in practitioner-orientated papers like O'Kane and Turnbull (2003).

C.1 Fitting the CDS Curve Using a OU Process for the Hazard Rate

It is well known (see for instance Brigo and Mercurio, 2006, pg. 699) that when the hazard rates are stochastic then the survival probability up to a time t is given by

$$SP(t) = \mathbb{E}_0\left[\exp\left(-\int_0^t \lambda_s \mathrm{d}s\right)\right]. \tag{10}$$

[4]Please note that Actual/360 is the day count convention and the number of days in one month may differ from month to month

When the hazard rate follows an OU process such as that in (4) we can calculate the expectation in closed form as

$$SP(t) = \exp[a(t) + b(t)\lambda_0] \tag{11}$$

with

$$a(t) = -\frac{(b(t)+t)(\alpha k - \frac{\sigma^2}{2})}{\alpha^2} - \frac{\sigma^2}{4\alpha} b(t)^2; \qquad b(t) = \frac{\mathrm{e}^{-\alpha t} - 1}{\alpha}. \tag{12}$$

See also Vasicek (1977); Luciano and Vigna (2006).

Note that the above equation automatically satisfies the initial condition $SP(0) = 1$. There are four parameters to calibrate k, α, σ and λ_0 and in order to calibrate the Vasicek model for the credit spreads we need only four points on the survival probability curve or, equivalently, four CDS spread values, for the same obligor.

Since there may be more than four values on the survival curve we propose to estimate the obligor individual parameters by minimising the residual error between the model implied SP values in (11) and the values obtained from the CDS market.

C.2 Piecewise Constant Hazard Rates

The survival probabilities can also be bootstrapped from

$$S(0, s_n = T) = \frac{(1-R)\sum_{i=1}^{n} DF(s_i)[SP(s_{i-1}) - SP(s_i)]}{\text{RiskyPV01}(0, s_n)} \tag{13}$$

considering the recovery rate R fixed. However, when there are less maturities for traded contracts than the entire set of time points for which survival probabilities must be calculated, this commonly used method does not work. A solution to this shortcoming can be bootstrapping the survival probabilities from the hazard rates curve as proposed by O'Kane and Turnbull (2003). The idea is to assume that the hazard rate curve be piecewise constant. Suppose the CMCDS contract we are interested in is traded at time t_v and we have CDS maturing at $T_1, \ldots, T_M$, $M > 1$.

With the convention $\lambda_1 = \lambda_{0,T_1}$, $\lambda_i = \lambda_{T_{i-1},T_i}$, $i = 2, \ldots, M$, for $\tau = T - t_v$ the function $SP(\tau)$ is assumed to satisfy

$$\begin{aligned} -\log SP(\tau) = \lambda_1 \tau \mathbb{1}_{[0,T_1)}(\tau) &+ \sum_{i=1}^{M-2} \left[\sum_{j=1}^{i} (\lambda_j - \lambda_{j+1}) T_j + \lambda_{i+1}\tau \right] \mathbb{1}_{[T_i, T_{i+1})}(\tau) \\ &+ \left[\sum_{j=1}^{M-1} (\lambda_j - \lambda_{j+1}) T_j + \lambda_{i+1}\tau \right] \mathbb{1}_{[T_{M-1},\infty)}(\tau) \end{aligned} \tag{14}$$

We use a monthly time grid $0 = t_0 < t_1 < \ldots < t_n = T$ with $t_i = i/12$ for each $n \in \mathcal{M}$, $\mathcal{M}$ being the set of available maturities in months, and assume that the premium is paid quarterly. Hence, for each $n \in \mathcal{M}$, using a numerical searching algorithm, we solve iteratively the equations

$$\frac{1}{2}\frac{S(0,\frac{n}{12})}{1-R}\sum_{j=0}^{\frac{n}{3}-1}\Delta(t_{n-3j},t_{n-3(j+1)})DF(t_{n-3j})[SP(t_{n-3(j+1)})+SP(t_{n-3j})]$$
$$=\sum_{i=1}^{n}DF(t_i)[SP(t_{i-1})-SP(t_i)] \quad (15)$$

for λ_i, $i=1,\dots,M$. Note that the pro-rata payment due to settlement date outside quarterly market calendar is not included here.

C.3 Calibration with Nelson-Siegel Interpolation

We assume here that the hazard rate is deterministic time-varying such that $\int_0^t \lambda(s)\mathrm{d}s = \Psi(t)t$. The role of function $\Psi(t)$ is to capture any term structure variation. One of the common choices for function $\Psi(t)$ is the Nelson-Siegel (see Nelson and Siegel, 1987) function[5]

$$\Psi(t)=\alpha_0+(\alpha_1+\alpha_2)\left(\frac{1-\exp(-\frac{t}{\alpha_3})}{\frac{t}{\alpha_3}}\right)-\alpha_2\exp\left(-\frac{t}{\alpha_3}\right) \quad (16)$$

This function can generate all sorts of curve shapes. The parameter α_0 is the long term mean of the default intensity. Parameter α_1 is the deviation from the mean, with $\alpha_1>0$ implying a downward sloping intensity and $\alpha_1<0$ implying an upward sloping term structure. In addition the reversion rate toward the long-term mean is negatively related to $\alpha_3>0$. The parameter α_2 is responsible for generating humps when it is different than zero. In practice Bluhm et al. (2003) advocate not using humps as this may lead to overfitting problems. This means that here we shall assume that $\alpha_2=0$ and estimate $\alpha_0,\alpha_1,\alpha_3$ from CDS spread data. The survival function (10) implies

$$SP(t)=\exp\left[-\left[\alpha_0+\alpha_1\left(\frac{1-\exp(-\frac{t}{\alpha_3})}{\frac{t}{\alpha_3}}\right)\right]t\right] \quad (17)$$

The CDS pricing equation are then used to estimate the parameters from CDS spread data using a nonlinear optimization algorithm for a suitable minimization function such as sum of squared errors or sum of absolute errors. This allows to compute the theoretical CDS premia.

II Recap of the Algorithm and Numerical Examples

In calculating the CMCDS premium the following steps are followed daily

1. Determine the Libor-swap discount curve
2. For each name build the survival probabilities with piecewise constant hazard rates, the ones from the Nelson-Siegel interpolation or the OU process

[5]Markit Partners, the leading data provider from which we obtained our data, are using a similar approach based on Nelson-Siegel interpolation to produce theoretical credit curves in the situations where liquidity of data is very low.

3. For each name calculate the numerical values of corresponding RiskyPV01 using (9)

4. Calculate the entire family of CDS forward curves using (3)

5. Calculate the participation rate PR using (8)

A Numerical Examples

As an illustration of the methods of the previous section to bootstrap the survival probabilities we consider for three obligors the term structure of CDS spreads on a particular day. In order to have

a. a firm with big CDS premia and all maturities available

b. a firm with small CDS premia and few maturities available

c. a firm with small CDS premia and all maturities available

we take into account the firms Abitibi Consol Inc, Microsoft Corp and Tesco PLC a use their CDS quotes on October 3rd 2005 (see Table I).

Table I: CDS spreads – October 3rd 2005.

Maturity (Months)	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
6	0.0128		0.0006
12	0.0165		0.0007
24	0.0233	0.0005	0.0011
36	0.0300	0.0005	0.0015
48	0.0350		0.0019
60	0.0394	0.0005	0.0023
84	0.0425		0.0032
120	0.0453	0.0009	0.0043
180	0.0454		0.0044
240	0.0458	0.0011	0.0044
360	0.0469		0.0048
R	0.3929	0.4	0.394

Example 1: NS+Piecewise Constant HR. We report the $\boldsymbol{\alpha}$ and $\hat{\boldsymbol{\alpha}}_w$ estimates in Table II along with the participation rates for a contract with maturity $T = 5$ years written against a reference spread with maturity $m = 5$ years. We also plot in Figure 2 the corresponding survival probabilities and the ones bootstrapped using the method from the previous section along with the theoretical and observed CDS spreads.

Table II: $\boldsymbol{\alpha}$ Estimates and participation rates. Panel A reports $\hat{\boldsymbol{\alpha}}$ and Panel B $\hat{\boldsymbol{\alpha}}_w$. Panel C shows the participation rates for a CMCDS with $T = m = 5$ using the bootstrapping procedure (row lambda), the Nelson-Siegel interpolation (NS) and the NS interpolation with weights in the objective function (NS w)

Panel A

	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
$\hat{\alpha}_0$	0.007039	0.001153	0.084241
$\hat{\alpha}_1$	-0.02284	0	-0.1783
$\hat{\alpha}_3$	0.162935	0.153387	0.099982

Panel B

	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
$\hat{\alpha}_0$	0.005082	—	0.079385
$\hat{\alpha}_1$	-0.01392	—	-0.14482
$\hat{\alpha}_3$	0.141648	—	0.078568

Panel C

PR	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
NS	0.780191	0.739188	0.544214
NS w	0.826868	—	0.752114
lambda	0.773298	0.589349	0.519317

Table III: θ estimates (Panel A) and participation rates with and without convexity adjustment (Panel B).

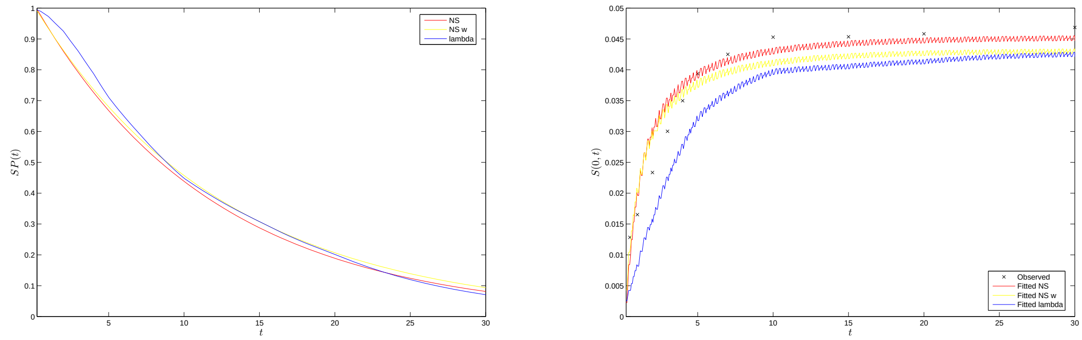
Panel A

	Abitibi	Microsoft	Tesco
k	0.038	0.0037	0.004
α	0.3749	2.6806	0.2919
λ_0	0.0498	0	0.0001
σ	0.0087	0	0.026

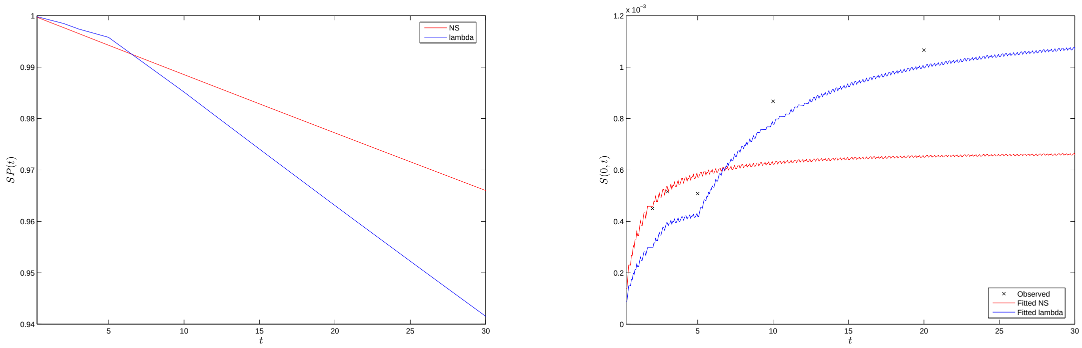
Panel B

PR	Abitibi	Microsoft	Tesco
With C.A.	0.7415	0.6163	0.6493
Without C.A.	0.7398	0.6163	0.4998

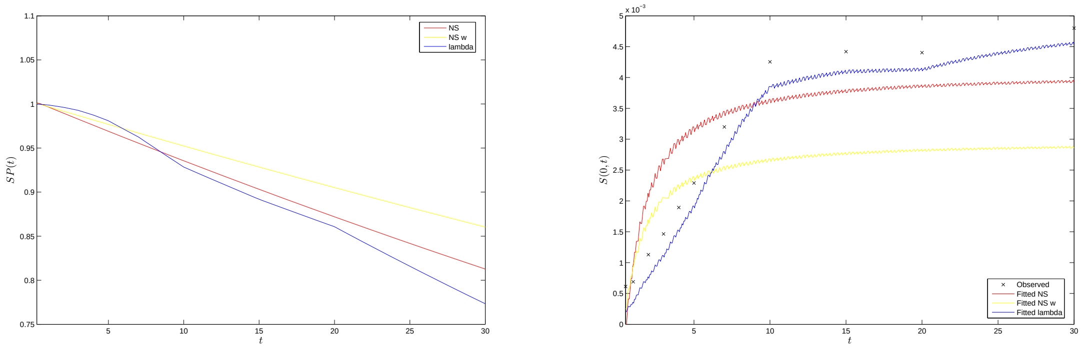
Figure 2: Bootstrapped Survival Probabilities and CDS spreads for Abitibi Consol Inc, Microsoft Corp and Tesco PLC, October 3rd 2005.



(a) Abitibi Consol Inc: Bootstrapped Survival Probabilities (left) and CDS spreads (right)

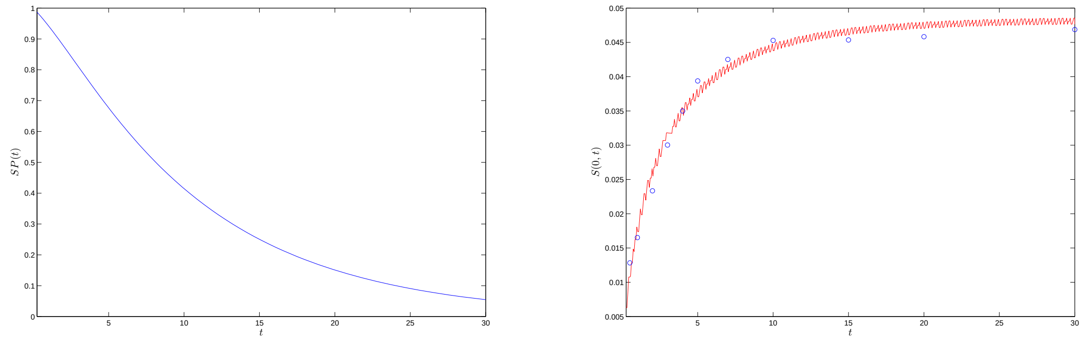


(b) Microsoft Corp: Bootstrapped Survival Probabilities (left) and CDS spreads (right)

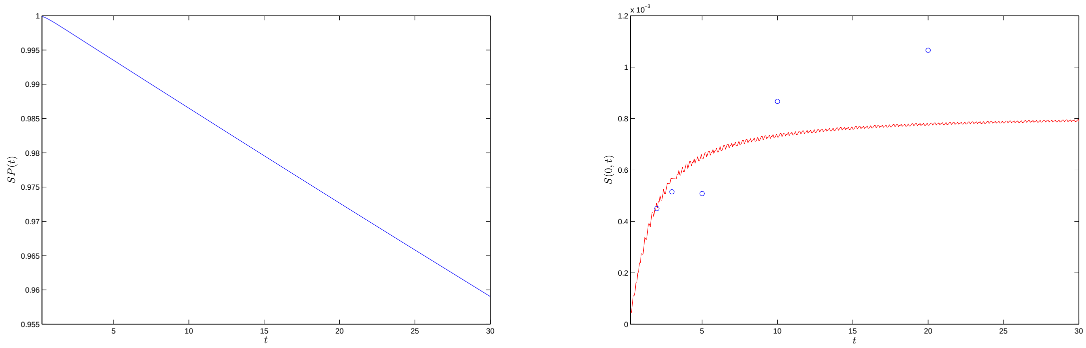


(c) Tesco PLC Bootstrapped Survival Probabilities (left) and CDS spreads (right)

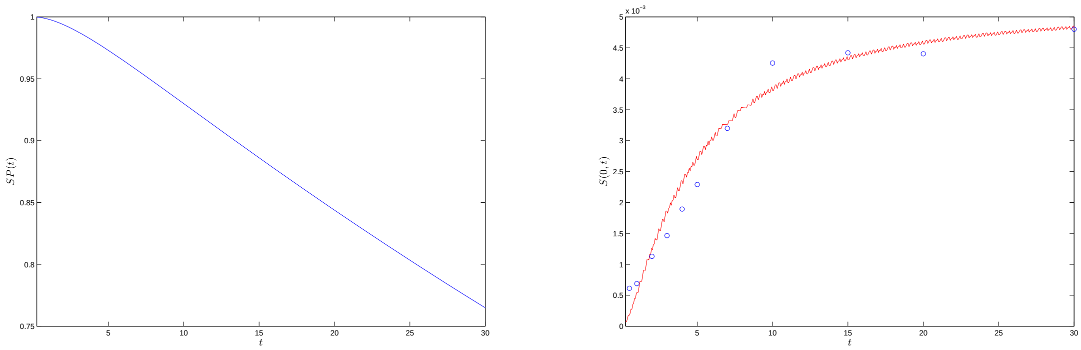
Figure 3: Bootstrapped Survival Probabilities and CDS spreads for Abitibi Consol Inc, Microsoft Corp and Tesco PLC using the OU process, October 3rd 2005.



(a) Abitibi Consol Inc: Bootstrapped Survival Probabilities (left) and CDS spreads (right)



(b) Microsoft Corp: Bootstrapped Survival Probabilities (left) and CDS spreads (right)



(c) Tesco PLC Bootstrapped Survival Probabilities (left) and CDS spreads (right)

Example 2: OU + Convexity adjustment. Here we assume the OU process for the default intensity. We want also to evaluate the impact of the convexity adjustment on the participation rate. Theoretical CDS and observed CDS are plotted in Figure 3. Notice that, given the limited number of observations for Microsoft, the fit is quite poor and this results in $\hat{\sigma} = 0$. The estimated parameters and participation rates are reported in Table III.

The general message coming from the two example is that the method with piecewise constant hazard rate of O'Kane and Turnbull (2003) seems to capture the shape of the CDS term structure even when the number of observation is limited. The downside, however, is that this procedure seems to consistently underestimate the fitted CDS. Further, the method is much slower than the other two (Nelson-Siegel and OU). As it appears clear from the pictures, the two parametric methods offer a very good fit when enough observations are available.

III Arbitrage Evidence in Credit Markets

In what follows we first describe the dataset and present an analysis based on daily marked-to-market of the trading strategy CMCDS-CDS for the two most liquid companies in the dataset. Next we study trading strategies based on the swap CDS-CMCDS to understand whether it would have been more profitable to sell CDS and to buy CMCDS on the same name or viceversa, during the five-years time period 2001–2006.

A Data Description

Our dataset consists of daily single-name composite spreads for the period January 2001–November 2006 with maturities 6m, 1y, 2y, 3y, 4y, 5y, 7y, 10y, 15y, 20y, 30y.

The composite spread is the average spread for an instrument provided to Markit by its contributors[6] after prices and spreads failing the data quality tests have been removed from the sample set. The cleaning process includes testing for Stale, Flat curves and Outlying data[7]. There are 2243 companies in the dataset. In some cases there are missing values, especially for not very liquid maturities. For each day and for each name also a recovery rate is reported. Additional informations like sector, rating and country are reported as well.

The number of companies in each sector and rating category are displayed in Table IV.

In Figure 4 we report for each payment date (the 20th of March, June, September and December) the number of companies for which we have the 5-years CDS spread and the recovery rate.

As far as the construction of the discount factor is concerned, we use Libor rates with maturities 1 month to 11 months and swap rates with maturities 1y, 2y, 3y, 4y, 5y, 7y, 10y, 20y, 30y. Libor-Swap data spans the same interval we have CDS data for.

B Daily Marked-to-Market of the Trading Strategy CMCDS–CDS

Having derived the time series of participation rates for a specific name and a reference CDS contract with maturity m years (typically 5 years) we start to analyse the properties of the

[6]Markit only builds composites when there are at least three contributors to each composite.

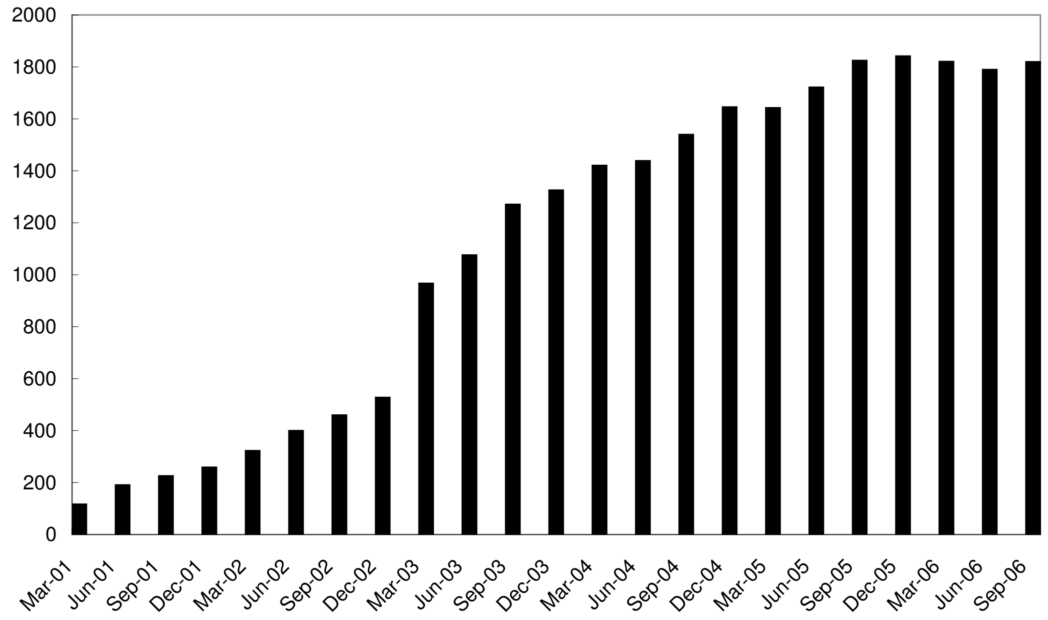
[7]On average Markit rejects approximately 45% of the CDS data received due to failure under any combination of the three criteria above.

Table IV: Number of companies available for each sector and rating. NA means that the rating is not available.

Sector	No of Companies
Basic Materials	156
Consumer Goods	229
Consumer Services	336
Financials	556
Government	66
Health Care	88
Industrials	279
Oil & Gas	131
Technology	80
Telecommunications	110
Utilities	212

Rating	No of Companies
AAA	59
AA	178
A	533
BBB	636
BB	285
B	234
CCC	56
D	4
NA	258

Figure 4: Number of companies for which we have the 5-years CDS spread and the recovery rate in each payment date (the 20th of March, June, September and December).



time series $\{y_t\}_{t=t_v,t_v+1,\ldots}$, where

$$y_t = \mathrm{PR}^{tv} \times S(t, t+m) - S(t_v, t_v + T),$$

t_v is the date we are interested in and PR^{tv} is the participation rate on that day.

This measure quantifies the daily marked-to-market of the trading strategy CMCDS-CDS that is settled at time t_v. The summary statistics of this time series will indicate whether overall, the floating leg in CMCDS is above or below the fixed CDS.

We express each y_t in basis points and consider two obligors whose CDS are very liquid, AT&T and Goldman Sachs Gp Inc. We assume $m = 5$ years and that the analysis is performed for the day January, 02 2003. In Table V we report the descriptive statistics for the series y computed using three approaches described before. The plots of the time series y and the empirical densities are reported in Figure 5.

It is obvious that the distribution of the difference in payments for the CMCDS relative to the CDS contract is most of the time negative indicating that for an investor it would have been convenient going short the CDS and long the CMCDS with reference entity AT&T and Goldman Sachs Gp Inc. Looking at the median for both this two companies, it follows that 50% of the time there were more than 35 bp between the floating rate in the CMCDS and the fixed rate in the single-name CDS referenced by the same name.

C Static Investment Analysis

In this first analysis we compute for all the companies for which we have the required data the profit (or loss) that an investor would have realized being long a CMCDS with maturity 5 years (the maturity of the reference CDS is 5 years as well) and short a CDS with both the contracts initiated on 20/09/2001. With this choice we can perform this analysis for a number

Table V: January, 02 2003: descriptive statistics for the time series y (basis points). NS is for Nelson Siegel, NSw is for Nelson Siegel weighted and lambda is for piece-wise constant hazard rates.

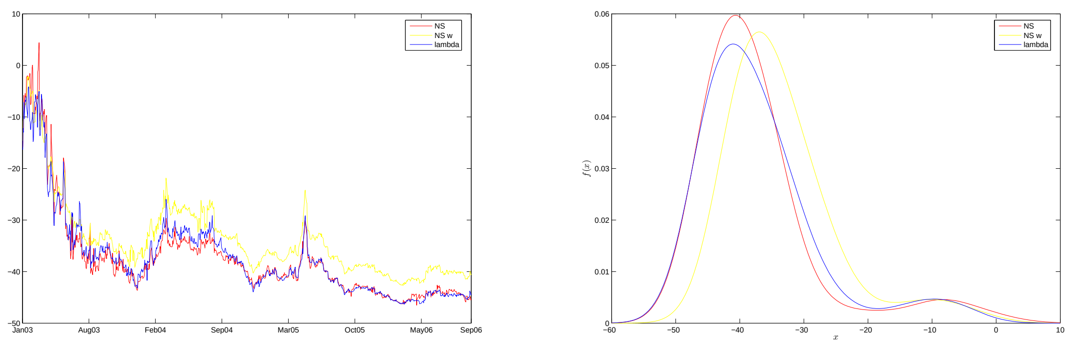
AT&T

	mean	median	std	min	max	95% Range	99% Range
NS	-37.2535	-39.6678	9.5306	-46.517	4.4133	39.3184	45.6049
NSw	-33.4328	-35.6085	8.2444	-42.7188	0.5009	34.4122	39.9939
lambda	-37.031	-39.1492	9.023	-46.3233	-2.5512	38.6349	40.9442

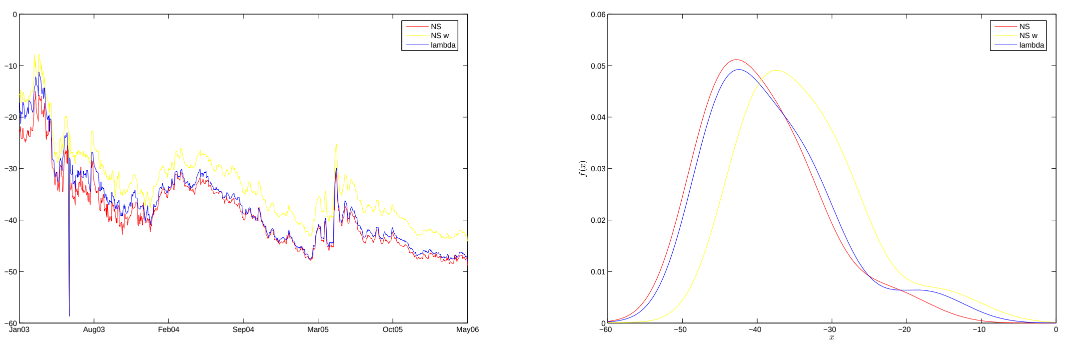
Goldman Sachs Gp Inc

	mean	median	std	min	max	95% Range	99% Range
NS	-39.2847	-40.6095	7.227	-58.6667	-15.0316	27.2412	32.3140
NS w	-33.7759	-35.1025	7.7681	-58.6667	-7.744	29.7924	34.9852
lambda	-37.9413	-39.6047	8.1287	-58.6667	-11.2436	31.4027	35.1944

Figure 5: Time series and Empirical Density for the time series y – January, 02 2003.



(a) AT&T: Times series y (left) and Empirical Density (right)

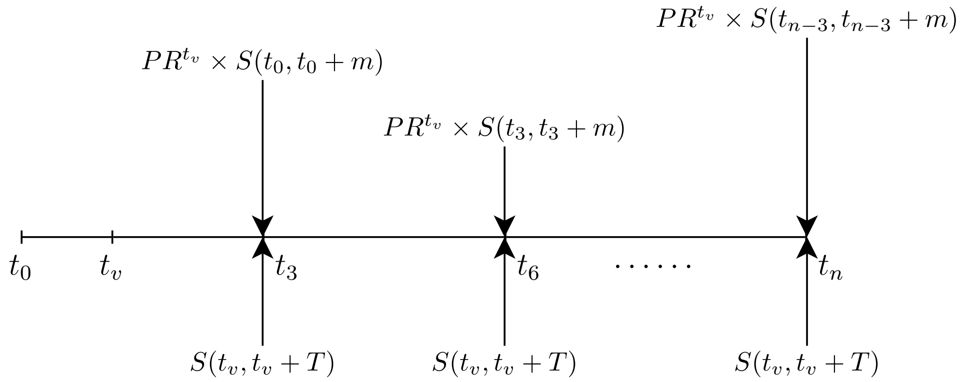


(b) Goldman Sachs Gp Inc: Times series y (left) and Empirical Density (right) of companies between 200 and 204, depending on the method used to compute the participation rate. In other words for $j = 1, 2, \ldots$ we compute the vector $\mathbf{z}$ with components[8]

$$z_j = \sum_{i=0}^{k-1} \Delta(t_{n-3i}, t_{n-3(i+1)}) \left[\mathrm{PR}_j^{t_v} \times S_j(t_{n-3(i+1)}, t_{n-3(i+1)} + m) - S_j(t_v, t_v + T) \right].$$

where, as usual t_i denotes a payment date, t_v is 20/09/2001 and $S_j(u, u+m)$ denotes the CDS spread at time u with maturity m for company j and $\mathrm{PR}_j^{t_v}$ is the participation rate[9] for company j at time t_v. An illustration is given in Figure 6.

Figure 6: Illustration of the statistical arbitrage analysis.



The results of this analysis are reported in Table VI, whereas the stem plots representing the z_j for the different methods are reported in Figure 7.

Table VI: Descriptive statistics for the vector of observations $\mathbf{z}$.

Method	Obs.	mean	median	std	min	max	95% Range	99% Range
Nelson Siegel	203	-172.344	-146.141	410.7125	-1450.21	1221.589	2102.138	2658.712
Piecewise Constant	204	-181.013	-156.096	382.8685	-1485.07	1121.221	2035.425	2515.842
OU process	200	-184.272	-157.282	408.5472	-1476.73	1366.107	2149.537	2816.082
OU process with c.a.	200	-165.075	-152.346	422.681	-1474.86	1366.107	2073.687	2815.146

We first compute the participation rate using the Nelson-Siegel interpolation. Using this method 173 observations out of 203 are negative (85.22%). This means that 85.22% of the companies had negative cumulative net payments with respect to an investment strategy swapping a CMCDS with a CDS. Next, we repeat the same analysis in the case in which the participation rate is calculated using the method which employs piecewise constant hazard rates. Using this

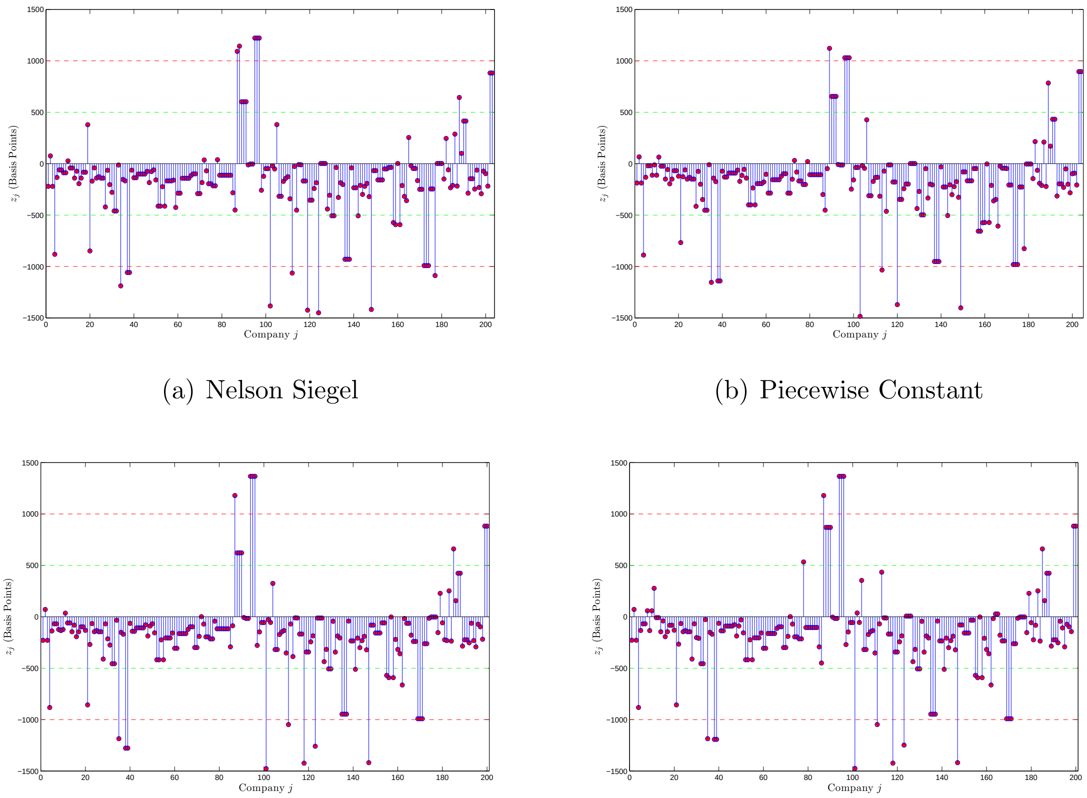
[8]Actually we compute the first term of the summation as follows:

$$\begin{aligned}
&\Delta(t_3, t_v) \mathbb{1}_{\{t_3 - t_v > 1mth\}} \left[\mathrm{PR}_j^{t_v} \times S_j(t_3, t_3 + m) - S_j(t_v, t_v + T) \right] \\
&+ \Delta(t_3, t_v) \mathbb{1}_{\{t_3 - t_v \le 1mth\}} \left[\mathrm{PR}_j^{t_v} \times S_j(t_6, t_6 + m) - S_j(t_v, t_v + T) \right] \\
&= \Delta(t_3, t_v) \left[\mathrm{PR}_j^{t_v} \left(S_j(t_3, t_3 + m) \mathbb{1}_{\{t_3 - t_v > 1mth\}} + S_j(t_6, t_6 + m) \mathbb{1}_{\{t_3 - t_v \le 1mth\}} \right) - S_j(t_v, t_v + T) \right]
\end{aligned}$$

to take into account the different behavior of the first coupon.

[9]To be precise this should read $\mathrm{PR}_j^{t_v}(m, T)$.

Figure 7: Vector of observations $\mathbf{z}$.



(a) Nelson Siegel

(b) Piecewise Constant

(c) OU process

(d) OU process with c.a.

method 181 observations out of 204 are negative (88.7%). When the participation rate is calculated assuming a OU process for the evolution of the hazard rate, we can see that the vector $\mathbf{z}$ has 181 negative components out of 200 (90.5%). Using this method we can also evaluate the impact of the convexity adjustment. In this case the vector $\mathbf{z}$ has 171 negative components out of 200 (85.5%).

From a portfolio point of view our analysis indicates that it would have been more profitable to sell CDS and buy CMCDS. In addition, as can be seen from the figure there are names for which it would have been profitable to buy single-name CDS and sell CMCDS with same maturity. Overall, it appears that there had been more names for which profits could have been made from selling single-name CDS and buying CMCDS.

To summarize the results of this first analysis, we report (Table VII) for each method how many companies have a z_j positive (negative), bigger than 500 bp (smaller than -500 bp) and bigger than 1000 bp (smaller than -1000 bp).

Table VII: This table reports for each method how many companies have a z_j positive (negative), bigger than 500 bp (smaller than -500 bp) and bigger than 1000 bp (smaller than -1000 bp).

	NS	lambda	OU	OU conv
pos	30	23	19	29
neg	173	181	181	171
> 500 bp	11	10	10	11
< -500 bp	23	23	23	23
> 1000 bp	5	4	4	4
< -1000 bp	9	7	8	8

It is interesting to sort the companies according to their z_j. First of all we report (Table VIII) a list of the companies with $z_j < -500$ bp and those with $z_j > 500$ bp for *all* the methods used. In Table IX we do the same for the -1000 and 1000 bp bounds.

In Table X we report for each method the five companies with the most negative z_j (Panel A) and the first five companies with the biggest z_j (Panel B).

According to three methods out of four (the exception is the Nelson-Siegel method) Hasbro Inc is the company for which the loss ($-z_j$) that an investor would have realized being long a CMCDS with maturity 5 years and short a CDS with both the contracts initiated on 20/09/2001 is maximum.

On the other hand using three methods (the exception being the one with piecewise constant hazard rates) Global Marine Inc is the company with maximum profit z_j. It is interesting to notice that every method include General Motors and Ford among the five companies with the biggest z_j.

Note that we also applied a cap on the floating payment and computed

$$z_j^{t_v,\mathrm{cap}} = \sum_{i=0}^{k-1} \Delta(t_{n-3i}, t_{n-3(i+1)}) \Big[\min\left\{800bp, \mathrm{PR}_j^{t_v} \times S_j(t_{n-3(i+1)}, t_{n-3(i+1)} + m)\right\} - S_j(t_v, t_v + T) \Big]$$

but the results are exactly the same.

Table VIII: The table reports the companies for which $z_j < -500$ bp according to all methods and those for which $z_j > 500$ bp according to all methods.

$z_j < -500$ bp	$z_j > 500$ bp
Aetna Inc.	Ford Mtr Co
Arrow Electrs Inc	Ford Mtr Cr Co
CNA Finl Corp	GA Pac Corp
Cap One Bk	GATX Finl Corp
Cap One Finl Corp	Gen Mtrs Corp
Hasbro Inc	Gillette Co
J C Penney Co Inc	Global Marine Inc
LA Pac Corp	Toys R Us Inc
Motorola Inc	Williams Cos Inc
NOVA Chems Corp	Wyeth
Nabors Inds Inc	
Nordstrom Inc	
Pennzoil Quaker St Co	
Raytheon Co	
Reebok Intl Ltd	
Roche Hldgs Inc	
ServiceMaster Co	
Shaw Comms Inc	
Sherwin Williams Co	

Table IX: The table reports the companies for which $z_j < -1000$ bp according to all methods. and those for which $z_j > 1000$ bp according to all methods.

$z_j < -1000$ bp	$z_j > 1000$ bp
CNA Finl Corp	Ford Mtr Co
Cap One Bk	Gen Mtrs Corp
Cap One Finl Corp	Gillette Co
Hasbro Inc	Global Marine Inc
J C Penney Co Inc	
LA Pac Corp	
Pennzoil Quaker St Co	

Table X: Panel A reports for each method the five companies with the most negative z_j. Panel B reports for each method the five companies with the biggest z_j.

Panel A

NS	lambda	OU	OU conv
Lucent Tech Inc	Hasbro Inc	Hasbro Inc	Hasbro Inc
LA Pac Corp	Pennzoil Quaker St Co	LA Pac Corp	LA Pac Corp
Pennzoil Quaker St Co	LA Pac Corp	Pennzoil Quaker St Co	Pennzoil Quaker St Co
Hasbro Inc	CNA Finl Corp	Cap One Bk	Lucent Tech Inc
CNA Finl Corp	Cap One Bk	Cap One Finl Corp	Cap One Bk

Panel B

NS	lambda	OU	OU conv
Finl Sec Assurn Inc	Wyeth	Wyeth	Wyeth
Ford Mtr Co	Gen Mtrs Corp	Ford Mtr Co	Ford Mtr Co
Gen Mtrs Corp	Gillette Co	Gen Mtrs Corp	Gen Mtrs Corp
Gillette Co	Global Marine Inc	Gillette Co	Gillette Co
Global Marine Inc	Ford Mtr Co	Global Marine Inc	Global Marine Inc

D Dynamic Investment Analysis

In the analysis that follows for each company j we look at

$$z_j^{t_v} = \sum_{i=0}^{k-1} \Delta(t_{n-3i}, t_{n-3(i+1)}) \left[\mathrm{PR}_j^{t_v} \times S_j(t_{n-3(i+1)}, t_{n-3(i+1)} + m) - S_j(t_v, t_v + T) \right]$$

for each day t_v between 02/01/2001 and 19/12/2001 for which we have the data required. Given company j, we denote by n_j the number of days t_v for which we can derive $z_j^{t_v}$.

The output of the present analysis is the vector $\bar{\mathbf{z}}$ with elements

$$\bar{z}_j = \frac{1}{n_j} \sum_{t_v=1}^{n_j} z_j^{t_v}.$$

In other words the yardstick measure for comparison is the average cumulative of net payments over a hypothetical portfolio strategy that could have been traded between 02/01/2001 and 19/12/2001. Hence each swap strategy CMCDS vs CDS that starts on any day within the above period is followed through maturity and the P/L is calculated and reported comparatively on an average basis.

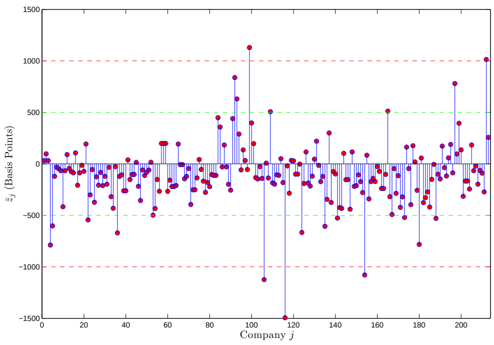
In Table XI descriptive statistics for the vector of observations $\bar{\mathbf{z}}$ for the four methods implemented are reported. Also a graphical representation of the $\bar{z}_j$ corresponding to the different obligors for the four methods is provided (Figure 8).

Using the Nelson-Siegel interpolation to compute the participation rate 160 observations out of 213 are negative (75.12%). When we use the method of the piecewise constant hazard rates 183 observations out of 213 are negative (85.92%). Next, we compute the participation rate assuming a OU process for the evolution of the hazard rate. In this case the negative components in the vector $\bar{\mathbf{z}}$ are 187 out of 213 (87.79%). Finally, using this method we take into account the impact of the convexity adjustment. In this case the vector $\bar{\mathbf{z}}$ has 155 negative

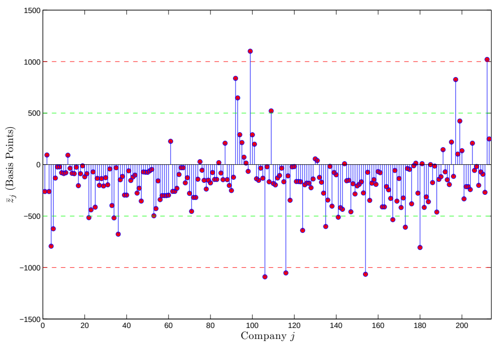
Table XI: Descriptive statistics for the vector of observations $\bar{\mathbf{z}}$.

Method	Obs.	mean	median	std	min	max	95% Range	99% Range
Nelson Siegel	213	-109.54	-112.407	298.6551	-1494.8	1129.924	1222.977	2349.756
Piecewise Constant	213	-153.432	-146.417	274.2414	-1090.6	1100.494	1239.63	2131.705
OU process	213	-169.898	-157.106	277.3286	-1475.95	1048.465	1191.47	2310.164
OU process with c.a.	213	-40.6761	-76.3784	445.3035	-1294.75	2592.18	1947.012	3505.057

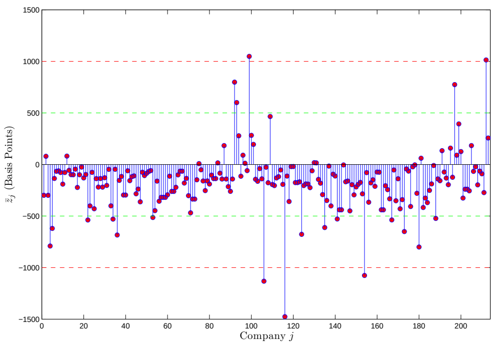
Figure 8: Vector of observations $\bar{\mathbf{z}}$.



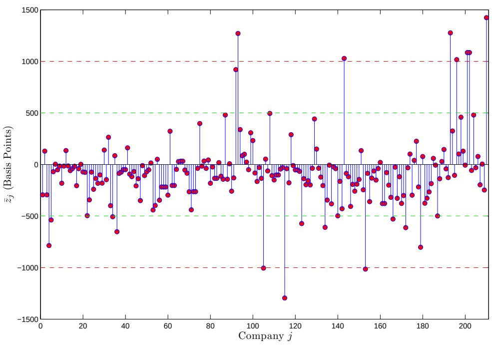
(a) Nelson Siegel



(b) Piecewise Constant



(c) OU process



(d) OU process with c.a.

components out of 213 (72.77%). Once again, the conclusion is that on average an investor could have been made profits by selling single-name CDS and buying CMCDS during the period analized.

As a summary of the results of this analysis we include Table XII, in which for each method are reported how many companies have a $\bar{z}_j$ positive (negative), bigger than 500 bp (smaller than -500 bp) and bigger than 1000 bp (smaller than -1000 bp).

Table XII: This table reports for each method how many companies have a $\bar{z}_j$ positive (negative), bigger than 500 bp (smaller than -500 bp) and bigger than 1000 bp (smaller than -1000 bp).

	NS	lambda	OU	OU conv
pos	53	30	26	58
neg	160	183	187	155
> 500 bp	7	6	5	11
< -500 bp	13	14	16	12
> 1000 bp	2	2	2	10
< -1000 bp	3	3	3	3

In Table XIII we report a list of the companies with $\bar{z}_j < -500$ bp and those with $\bar{z}_j > 500$ bp for *all* the methods used. In Table XIV we do the same for the -1000 and 1000 bp bounds.

Table XIII: The table reports the companies for which $\bar{z}_j < -500$ bp according to all methods and those for which $\bar{z}_j > 500$ bp according to all methods.

$\bar{z}_j < -500$ bp	$\bar{z}_j > 500$ bp
Aetna Inc.	Ford Mtr Co
Agrium Inc	Ford Mtr Cr Co
CNA Finl Corp	Gen Mtrs Corp
Hasbro Inc	Toys R Us Inc
J C Penney Co Inc	Williams Cos Inc
LA Pac Corp	
Mattel Inc	
Pennzoil Quaker St Co	
SUPERVALU INC	
ServiceMaster Co	

In Table XV we report for each method the five companies with the most negative $\bar{z}_j$ (Panel A) and the first five companies with the biggest $\bar{z}_j$ (Panel B).

IV Conclusions and Further Developments

In this study a large database of single-name CDS premia has been used to produce the corresponding CMCDS prices. In order to derive the participation rate needed to calculate the

Table XIV: The table reports the companies for which $\bar{z}_j < -1000$ bp according to all methodsand those for which $\bar{z}_j > 1000$ bp according to all methods.

$\bar{z}_j < -1000$ bp	$\bar{z}_j > 1000$ bp
Hasbro Inc	Gen Mtrs Corp
J C Penney Co Inc	Williams Cos Inc
Pennzoil Quaker St Co	

Table XV: Panel A reports for each method the five companies with the most negative $\bar{z}_j$. Panel B reports for each method the five companies with the biggest $\bar{z}_j$.

Panel A

NS	lambda	OU	OU conv
J C Penney Co Inc	Hasbro Inc	J C Penney Co Inc	J C Penney Co Inc
Hasbro Inc	Pennzoil Quaker St Co	Hasbro Inc	Pennzoil Quaker St Co
Pennzoil Quaker St Co	J C Penney Co Inc	Pennzoil Quaker St Co	Hasbro Inc
Aetna Inc.	ServiceMaster Co	ServiceMaster Co	ServiceMaster Co
ServiceMaster Co	Aetna Inc.	Aetna Inc.	Aetna Inc.

Panel B

NS	lambda	OU	OU conv
Ford Mtr Cr Co	Ford Mtr Cr Co	Ford Mtr Cr Co	Textron Inc
Toys R Us Inc	Toys R Us Inc	Toys R Us Inc	Wyeth
Ford Mtr Co	Ford Mtr Co	Ford Mtr Co	Gen Mtrs Corp
Williams Cos Inc	Williams Cos Inc	Williams Cos Inc	Williams Cos Inc
Gen Mtrs Corp	Gen Mtrs Corp	Gen Mtrs Corp	Wells Fargo & Co

CMCDS prices, we implemented both parametric (the Nelson-Siegel interpolation and the hazard rates described by an Ornstein-Uhlenbeck process) and nonparametric methods (piecewise constant hazard rates). For each day and for each name all these methods utilize the term structure of single-name CDS prices along with information regarding the recovery rate and the discount factors bootstrapped from the Libor/Swap rates to return the corresponding participation rate. This allowed us to build a database of single-name CMCDS premia that was used to identify possible imbalances that may exist in the credit markets when pairing CDS and CMCDS on the same name. The general idea is to form a swap type of trading strategy whereby a fixed premium payment is netted against a floating one, both representing protection premia against default. This strategy has the advantage that default risk is eliminated and only counterparty risk is taken. Therefore we have computed for all the companies for which we have the required data the profit (or loss) that an investor would have realized being long a CMCDS with maturity 5 years and short a CDS with both the contracts initiated on 20/09/2001. Then we have done the same analysis for a contract initiated between the beginning of our sample and 01/11/2001. In both cases and for each method implemented we have reported for how many companies the above strategy produces a gain or a loss bigger than 500 or 1000 bp. It appears that, in general, it would have been more profitable to sell CDS and to buy CMCDS. Considering all the methods implemented, at least 85% of the names analysed had a negative cumulative net trading profit/loss over the 5 years period considered. Also when a convexity adjustment has been taken into account, we reached substancially the same conclusion. The method which involves convexity adjustment seems only to reduce the loss that an investor would have incurred in by buying CDS and selling CMCDS over the 5 years period considered. The percentage of names for which the above strategy would have led to a loss, however, does not change dramatically. Also a cap on the floating payment was introduced, but this did not change the results. We have also reported the names for which the strategy gives a gain or a loss bigger than 500 or 1000 bp no matter what the method used to compute the participation rate. Interestingly the strategy involving the company "Ford Mtr Co" leads to a profit bigger than 1000 bp in the first analysis and between 500 bp and 1000 bp for the second analysis. On the other hand, for both the analysis considered, the trading strategy involving the company "Gen Mtrs Corp" leads to a profit bigger than 1000 bp for all the methods implemented. In general, these two companies appear among those with the biggest profit, in both the first and the second statistical arbitrage analysis.

Many extension of this work are possible. First of all, it could be very interesting to solve the problem of the convexity adjustment by evaluating in a lattice payment and loss leg and hence the participation rate. Since so far only one method which allows for the convexity adjustment has been identified and implemented, another method will make the results more robust and the conclusions regarding the trading strategies stronger. Of course there are many issue to consider in these approach, for instance the choice of the process for the hazard rate, the calibration of the tree and the parameter estimation. Some other issues that could be investigated in these research include the effect of the different maturities of the CMCDS, the impact of rating and sector category and the impact of the convexity adjustment, which should be bigger for longer maturities.

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