NON-UNIQUENESS IN THE FIRST GENERATION BALANCE OF PAYMENTS CRISIS MODELS

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Abstract

In this paper I show that an attack on the Central Bank's foreign exchange reserves can take place on any date in a "first generation" balance of payments crisis model, contrary to Krugman (1979). This may leave the Central Bank with reserves in excess of the level that they wish to defend, which seems consistent with the data. But for the attack to be successful the amount of foreign exchange reserves that the Central Bank will lose on any date is uniquely determined.

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1. INTRODUCTION

Krugman's modelling of a balance of payments crisis—referred to as the “first generation” crisis models-- has been one of the most influential ideas in open economy macroeconomics in the recent past. It has now found its way into undergraduate text–books also.1 Krugman (1979), building on Salant and Henderson (1978), showed that for a small open economy following policies inconsistent with the fixed exchange rate regime it is on, there is a precise moment when the fixed exchange rate regime would be abandoned. Neither was this moment of switch when everyone realises that the fixed exchange rate regime will ultimately collapse, nor when the Central Bank has lost all its foreign exchange reserves and is unable to maintain a fixed exchange rate regime. Krugman showed that the Central Bank would lose its remaining stock of foreign exchange reserves *discretely* when the domestic credit component of the money supply reaches the level equal to the nominal money stock required under a flexible exchange rate regime.[2] The exchange rate then depreciates in a continuous manner --i.e., without a discrete jump--from its fixed rate value.[3]

The unique date of the crisis can be explained clearly using the notion of a shadow floating exchange rate. This is the exchange rate that would prevail on any date if the economy were to move to a flexible exchange rate regime on that date. Since by perfect foresight anticipated jumps in the exchange rate are ruled out, the attack on the Central Bank's reserves occurs when the shadow floating exchange rate just equals the fixed rate (or differ by the amount of any costs of transaction).

In this paper, I show that in the Krugman model there are multiple equilibria. The Central Bank may be in possession of a lot of foreign exchange reserves but not an "adequate amount" i.e., not enough to maintain a fixed exchange rate. The reason that I propose for the existence of multiple equilibria is different from some recent attempts to generate multiplicity of equilibria in the Krugman model. These rely on some feedback mechanism which moves the shadow

1 See Krugman and Obstfeld (2000), pp. 526-529.

2 On the assumptions that the level of reserves that the Central Bank will defend is zero and following the collapse of the peg the economy will move to a flexible exchange rate system.

3 Throughout we will consider bubble-free solutions.

exchange rate path (see e.g., Flood and Marion (1998), Cavallari and Corsetti (2000)). In these models when an attack occurs, the Central Bank loses all its foreign exchange reserves (above what it "defends"). In my analysis the public may generate an attack on any date which may leave the Central Bank with more reserves than it wants to defend. There are multiple paths for the shadow exchange rate. While the crisis can occur on any date, the amount of reserves that the Central Bank loses on the day of the crisis is exactly equal to the one predicted by Krugman.4 My analysis actually helps reconcile an apparent anomaly cited by Obstfeld and Rogoff (1996): namely that in actual fact most Central Banks that were attacked had sizeable foreign exchange reserves.

In section 2, I outline the basic Krugman model shorn of the inessentials. In section 3, I show that there are an infinity of dates on which the fixed rate could collapse. Some concluding comments are offered in section 4.

2. THE MODEL

I use Dornbusch's (1987) exposition of the Krugman model since it is well-known territory.5

A small open economy produces and consumes a traded good. Hence purchasing-power-parity prevails at all times i.e.,

$$P(t) = E(t).P^*(t) \qquad (1)$$

P (resp. P*) is the domestic currency price (resp. foreign currency price) of the good and E is the exchange rate (the domestic currency price of foreign exchange). Setting the given value of P* to unity, the domestic price level is equal to the exchange rate.

Uncovered interest parity is assumed to hold between domestic and foreign currency

4 This is equal to the difference between the fixed and flexible exchange rate money supplies.

5 See e.g., Garber and Svensson (1995), Agenor, Bhandari and Flood (1992), Blackburn and Sola (1993)). Flood and Marion (1998) also use an identical model but then add a risk premium.

assets i.e.,

$$i(t) = i^*(t) + \lambda(t) \qquad (2)$$

i (resp. i^*) is the domestic interest rate (resp. foreign interest rate) and λ is the expected rate of depreciation of the domestic currency. With perfect foresight, $\lambda(t) = d \log E(t)/dt$.

Full employment prevails and the level of output (y) is normalized to zero. Real money demand, therefore, can be expressed solely as a function of the nominal interest rate, i. Money market equilibrium requires that [6]

$$M(t)/E(t) = a - b.i(t) \qquad a, b > 0 \qquad (3)$$

M is the nominal money stock. In the absence of commercial banks we have

$$M(t) \equiv R(t) + D(t) \qquad (4)$$

R is the Central Bank's foreign exchange reserves (measured in domestic currency) and D the domestic credit component of the money supply.

In conformity with the first generation models of balance of payments crises, the domestic credit component is assumed to grow at a constant rate μ (e.g., because the government runs a budget deficit which the monetary authority passively monetises)[7]. D(t) is a continuous function of time except when the Central Bank conducts an open market operation.

$$dD(t)/dt = \mu.D(t) \qquad (5)$$

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6 Those who dislike ad-hoc macro models could derive the following equation from an infinite horizon model with money and consumption yielding utility. We get (3) by inverting $v'(m) = i.u'(c)$ where the felicity function is additively separable in consumption c and real balances m. Finally c is constant through time (equal to y+i*F, where F is the net foreign asset position of the economy).

7 See De Kock and Grilli (1993) for a discussion of this in a general context.

The Central Bank faces an upward sloping supply for borrowing foreign exchange reserves so there are costs to borrowing reserves (see Buiter (1987)). Finally, there is a target level of foreign exchange reserves which the Central Bank will defend i.e., it is prepared to commit the rest of its holding of foreign reserves to the defence of the fixed exchange rate system. Following the literature, we assume that this target level $\tilde{R}$ is known.[8]

While the economy is on a fixed exchange rate system (with $E = \overline{E}$) and it is expected to remain on that system (with λ=0)

$$i = i^{*} \tag{6}$$

$$M^{F} / \overline{E} = a - b.i^{*} \equiv \alpha \tag{7}$$

M^{F} is the nominal money supply and α is the real money demand in the fixed rate regime.[9]

I assume, as is standard, that when the fixed rate system collapses, we move to a freely floating exchange rate regime.[10] Given the structure of the model the economy can jump to the flexible exchange rate steady state instantaneously with all the nominal variables growing at μ.

Under a floating rate, we thus have

$$M^{s}(t) / E(t) = a - b(i^{*} + \mu) \equiv \beta \tag{8}$$

$M^{s}(t)$ is the nominal money supply and β is the real money demand in the floating rate regime.

8 In a previous version of this I had assumed, like Krugman, that the level of reserves that the Central Bank will defend is zero. This normalisation turned out to be source of confusion for the point I am trying to make in this paper and hence has been jettisoned here.

9 It is evident from equation (7) the nominal money stock, which will generate equilibrium in the money market is constant under a fixed exchange rate .

10 Alternative post-collapse regimes have been discussed. See e.g., Agenor, Bhandari and Flood (1992) for a discussion of some of these.

From equations (7) and (8), note

$$\alpha - \beta = b\mu$$

It is easy to check that here a continuing increase in the domestic credit component of the money supply is inconsistent with an everlasting fixed exchange rate regime. In finite time, D will become larger than the fixed exchange rate level of money supply M^F.

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As long as the economy is on a fixed exchange rate, D is growing at a rate μ, and foreign reserves fall by μD since the sum of D and R is M^F i.e.,

$$dD(t)/dt = - dR(t)/dt = \mu.D(t) \qquad (9)$$

Krugman's insight was to point out that when domestic credit component reaches $M^s(T) - \tilde{R}$ say on date T (with the exchange rate at $\bar{E}$), there will be a discrete run on the Central Bank's reserves and the entire remaining stock of reserves equal to the difference between M^F and M^s(T)--call it ΔR(T)-- will be cleaned out, leaving it with $\tilde{R}$ level of reserves (remember that $\tilde{R}$ is the level which the Central Bank is assumed to defend). The exchange rate evolves continuously from $\bar{E}$ i.e., it depreciates at a rate μ from date T onwards (as do all other nominal variables). This is the bubble-free solution proposed by Krugman. Everyone expects the fixed rate system to collapse on T and it does--the chronicle of a death foretold.

To see this more clearly, look at figure 1. On the horizontal axis we have real money balances and on the vertical axis the nominal interest rate. The demand for real balances is the downward-sloping line LL. The fixed exchange rate equilibrium is shown at point A with real balances at OF^i and nominal interest rate at OI. This corresponds to the nominal money supply M^F. For convenience assume $\bar{E}$ = 1, by a suitable choice of units. This allows us to talk interchangeably between real and nominal money supply as long as the economy is on a fixed exchange rate system. In the diagram we show R measured from right to left (starting vertically below A), while D is measured from left to right starting from the origin. Since D is growing over time, while M^F is constant, R must be falling over time (see equation (9) above). When on date T, the level of D+$\tilde{R}$ reaches M^s--shown in figure 1 as the horizontal distance OF^l--or D reaches OU since $\tilde{R}$ is the distance UF^l, agents clean out the rest of the Central Bank's reserves (i.e., OF^i less OF^l) and the economy moves to a floating rate system with all nominal variables growing at μ and the nominal interest rate at i^*+μ (given by the vertical distance ON). This equilibrium is shown at point S in the diagram.

It is easy to show that, following Krugman, the regime shift has to occur at T and not earlier or later. For this one needs to remember that since there is no new information the exchange rate E has to move continuously--anticipated jumps in E are ruled out by arbitrage.[11]

To clarify we can use the notion of a shadow floating exchange rate. What would happen if the system was to move to a floating rate before T (by cleaning out the foreign exchange reserves in excess of $\tilde{R}$)--say when D has reached point V on date τ? Since D(τ)+$\tilde{R}$ is short of M^s, the exchange rate necessary for money market equilibrium and which will take the economy to the floating exchange rate steady-state--the shadow floating exchange rate X(τ)--would have to appreciate relative to its fixed rate value so that real balances are equal to β. The earlier the shift to a float, the larger is the appreciation. This is shown in figure 2. Agents would be making an *anticipated* capital loss by buying foreign exchange--for t≤T they would be buying at $\overline{E}$ but its market price is X(t) ($X(t) \leq \overline{E}$). Perfect foresight would rule out such an eventuality. Similarly for postponing the float beyond T -- here there are anticipated capital gains with the shadow rate lying above $\overline{E}$. At time T, the shadow floating rate is just equal to the fixed rate and hence that is the only possible time when the economy can move to a floating rate without a jump in the level of the exchange rate.

Formally

$$X(t) = (1/\beta).(D(t)+\tilde{R}) = (1/\beta).(D(0)\exp.(\mu t)+\tilde{R}) \qquad (10)$$

11 See Turnovsky (1995), Chapter 3 for a discussion of the continuity of rational expectations solutions.

and T is determined from X(T) = $\overline{E}$

$$T = [\log(\beta\overline{E} - \tilde{R}) - \log D(0)]/\mu \qquad (11)$$

and the loss of reserves at T

$$M^F - M^s = b\mu\overline{E} \qquad (12)$$

3. MULTIPLE EQUILIBRIA

I now proceed to show that T is not uniquely determined and, at best, T is the maximum of all possible times of attack if indeed it is associated with the minimum level of reserves that the Central Bank will defend.[12] Two ingredients are required for this: first, one has to remember that the money supply is demand determined in a fixed exchange rate world with uncovered interest parity; and second, that on the date of the crisis agents should corner $b.\mu.\overline{E}$ amount of foreign exchange reserves (i.e., the amount that the public attacks on date T in Krugman's model).

To motivate my argument (but only to motivate), suppose that the private agents do not know that the Central Bank will defend $\tilde{R}$ level of reserves, but believe (with point certainty—and there is no disagreement among the agents, as is true in a representative agent framework[13]) that it will defend $R' > \tilde{R}$ (given by distance VF^l in figure 1) of reserves.[14] So when D reaches the value $M^s - R'$ (distance OV in figure 1) they will proceed to clean out $b\mu\overline{E}$ ($F^l F^i$ in figure 1) leaving it with R'. The economy moves to a floating rate regime after this. No discrete jump in the shadow exchange rate is called for! Here the Central Bank has reserves above what it

[12] As will become evident the cause of multiple equilibria in my model does not depend on any different behaviour on the part of the Central Bank as in Flood and Marion's attempt at marrying the first and second generation models (see Flood and Marion (1998) section 2.2), or Cavallri and Corsetti (2000).

13 So we are not in a Morris and Shin (1998) world.

14 Both the assumptions about point certainty and $R' > \tilde{R}$ are important. See Smith (2001) for a general discussion of uncertainty about the level that the Central Bank will defend.

would defend and at the margin it can sell foreign exchange at $\overline{E}$, but it does not have enough reserves to ensure money market equilibrium under fixed exchange rates. A viable price-fixing scheme has to meet market demand. At an interest rate i* (OI in figure 1), to sustain a fixed exchange regime, the Central Bank has to meet money demand equal to OF^i (which is now greater than the money supply OF^l). After the attack on the Central Bank's reserves, the interest rate will increase (and with perfect foresight this implies, λ=μ). This new level of M (OF^l) is consistent with a floating rate regime and E moving continuously. The belief about the target level of reserves (R') is self-fulfilling. For each level R' of reserves that the private sector believes that the Central Bank would defend there is a date of attack to corner $b\mu\overline{E}$ ($F^l F^i$) of foreign exchange reserves.

Now drop the assumption that agents do not know what level of reserves the Central Bank will defend. Suppose that every one knows that the authorities will in fact defend $\tilde{R}$. Is it possible that the private sector can still effect a shift to a flexible exchange rate system before $\tilde{R}$ has been reached? Take point V in figure 1. Money supply is backed by OV of D and VF^l plus $F^l F^i$ of R. Agents know the minimum level of foreign exchange reserves that the Central Bank will defend (which we have set equal to $\tilde{R}$). They clean out $F^l F^i$ of the reserves leaving it with VF^l . Again the economy moves to a floating rate regime after this. No discrete jump in the shadow exchange rate is called for! Here the Central Bank has reserves above what it would defend and at the margin it can sell foreign exchange at $\overline{E}$, but it does not have enough reserves to ensure money market equilibrium.

Another way of saying this is that for every level of foreign exchange reserves that the private agents leave the monetary authority with, there is a shadow exchange rate path. And on each date t, there is one shadow exchange rate path, which does not require a jump in the exchange rate in moving to a float. The earlier the economy moves to a float, the larger the reserves the Central Bank is left with. Using this logic one can shade the entire area above the shadow rate path shown in figure 2.(which is drawn for $\tilde{R}$, the minimum that the Central Bank will defend).

The foregoing discussion can be summarized as Proposition 1.

Proposition 1: On any date the public can force a collapse of the fixed exchange rate system without a discontinuous move in the shadow exchange as long as it attacks $b\mu\bar{E}$ units of foreign exchange reserves of the Central Bank.

With a fixed exchange rate and uncovered interest parity neither open market operations nor borrowing of foreign exchange reserves would help, since money supply is demand determined. To see this assume the Central Bank conducts an expansionary open market operation by buying domestic bonds from the public to try and restore the money supply to the fixed exchange rate level. It would lose foreign exchange reserves of an equivalent amount. If it persists in carrying out these open market operations it would lose all the foreign exchange reserves until $\tilde{R}$ is reached. Thus reserves would be driven down to the minimum level that the Central Bank would defend as in Krugman's example. But even here the attack could occur on any date. Multiple equilibria remain even when the Central Bank tries to play Canute.

Let us do a formal analysis similar to that in the previous section for this argument

$$X^M(t) = (1/\beta).[D(t) + R'(j)] = (1/\beta).[D(0)\exp.(\mu t) + R'(j)] \tag{13}$$

with the index j∈J such that R(0)>R(0)-ΔR(T^M)≥ R'(j)≥ $\tilde{R}$ ≥0 where R' is the amount of reserves the Central Bank is left with following the shift to a floating rate regime--which in section 2 above was $\tilde{R}$. X^M is the shadow floating rate when there are multiple equilibria.

Again setting $X^M(T^M)=\bar{E}$, we can solve for T^M

$$T^M = [(\log(\beta\tilde{E} - R'(j)) - logD(0)]/\mu \tag{14}$$

Comparing (14) with (11), we find that $T^M \leq T$ since R'(j) $\geq \tilde{R} \geq 0$. Also the larger is j and R'(j), the smaller is T^M.

A simple numerical example may clarify this further. Suppose as we had assumed above that $\overline{E} = 1$. Let $\tilde{R} = 0$, α=100 and β=60. In the Krugman case on date T, the Central Bank loses α-β=40 units of reserves and D(T)=60. But suppose on date τ, D(τ)=45 and $R(\tau^-)$= 55. If on date τ, the Central Bank loses 40 units of reserves (i.e., $R(\tau^+)$=15), then the money supply will be 60(≡D(τ)+$R(\tau^+)$), which at $\overline{E} = 1$ is exactly equal to β.

Note one implication of the above analysis, namely that if the fixed rate system is eventually going to break down there is no notion of "sufficient reserves" or borrowing to postpone the day of reckoning. The large holdings of foreign exchange reserves by countries whose Central Banks suffered attacks as reported in Table 8.1 (p. 566) of Obstfeld and Rogoff (1996) lends support to our result. Since only the amount of reserves that the private sector needs to corner is the difference between the fixed rate money supply and the flexible rate money supply (i.e., ΔR(T^M)), the actual holding of foreign exchange reserves by the Central Bank is irrelevant to the ability of the private sector to engineer the collapse. It is worth repeating that the amount of reserves that the public captures is the same (equal to the difference between the fixed and flexible exchange rate money demands) no matter when the attack occurs.

What determines the timing of the attack in this section? We are unable to answer this question within the minimalist framework of this model.[15] A feedback mechanism of the type discussed in Cavallari and Corsetti (2000) could be the cause. Or extraneous considerations like sunspots or model specific concerns of credibility could be introduced. I emphasise that the results have been obtained in a first generation model because of the postulated behaviour of the Central Bank. As in Krugman (1979), the Central Bank continues to "monetise" the deficit at a rate μ. The only new twist here is that the timing of the switch to a flexible exchange rate regime does not depend on the private sector cornering all of the Central Bank's reserves (in excess of the minimum amount they would defend).

4. CONCLUSIONS

[15] This is also true of most second generation models—these are models where the game between the Central Bank and private agents is explicitly modelled--with multiple equilibria--see e.g., Obstfeld (1996). Morris and Shin (1998) show how a unique equilibrium may be reached.

I showed the time of attack in Krugman's model, which results in a collapse of the fixed exchange rate system, is not uniquely determined. A continuum of paths to a floating regime exist i.e., the model has multiple equilibria. The only difference across periods (in which the attack can occur) is the amount of foreign exchange reserves left with the Central Bank and hence the proportion of foreign exchange reserves that will back the money supply under a floating rate regime. The Central Bank's holding of reserves or its ability to borrow, is not germane to the issue of collapse (as long as the fixed exchange rate is expected to collapse eventually). The Central Bank may hold "large" stocks of reserves but not "enough" to maintain a fixed rate regime. Its "large" reserves and domestic credit component of the money supply do not add up to the fixed exchange rate money supply, which is demand determined. In a price fixing scheme, the price-fixing authority has to meet the market demand at the price it has fixed-- ruling out rationing, of course-- otherwise its promise to fix the price is not credible.

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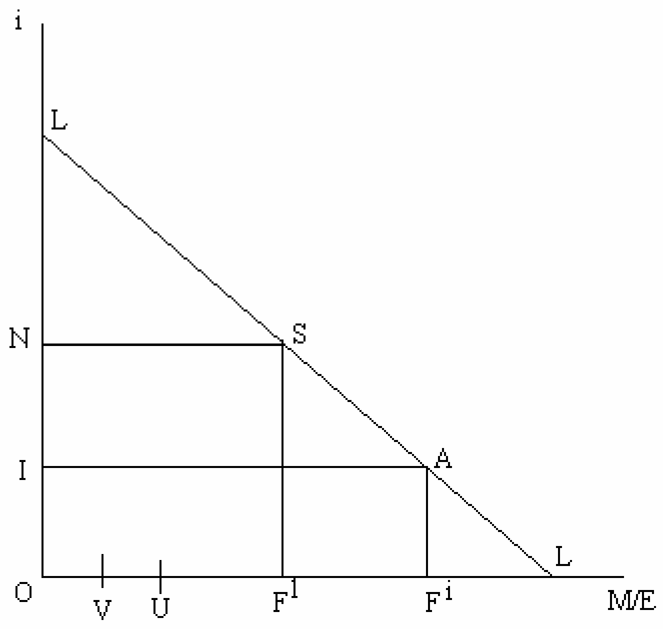

Figure 1

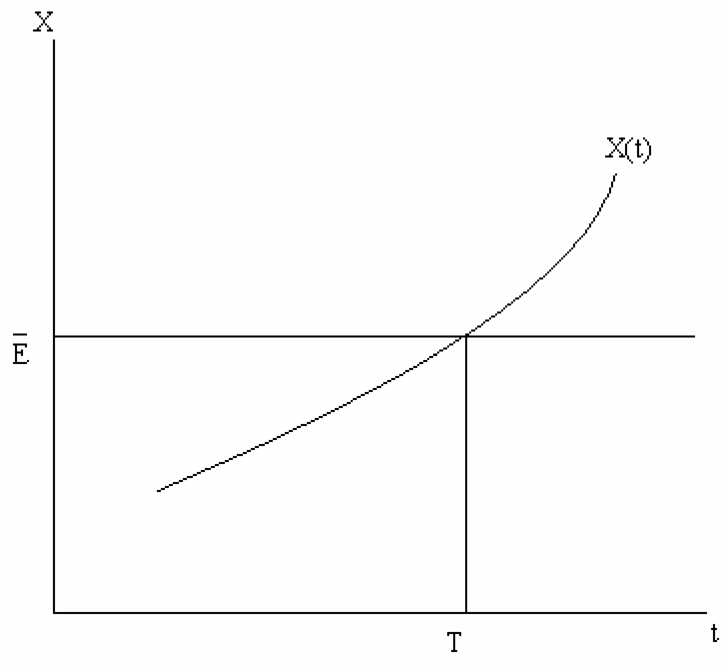

Figure 2